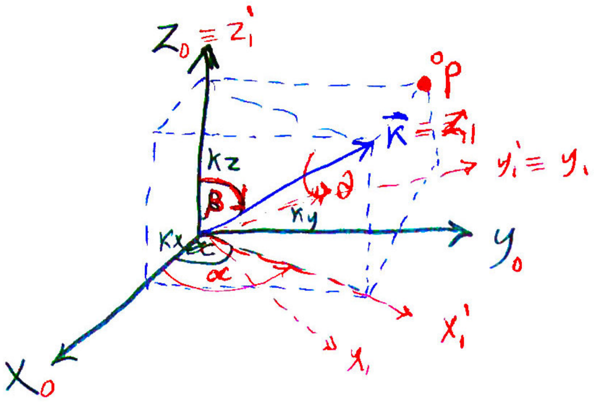
# Robotics

## Section 5

### Axis/Angle representation




$\vec{K}$ unit vector

- ويمكن تمثيله بمركبتين فقط ومنها نستنتج المركبة الثالثة

- ويمكن تمثيل الدوران في الفراغ من خلال unit vector $\vec{K}$ وزاوية الدوران حوله $\theta$

P: to be Rotated about K axis

→ Assume Frame 1 [any frame contain $\vec{K}$]

لا يشترط أن تكون $\vec{K}$ أحد المحاور الأساسية لـ frame

→ Assume Frame 1 Attached to $\vec{K}$

K in the direction of $Z_1$

given : $^0P \longrightarrow {}^1P$

$$^1P_{before} = {}^1R_0 \, {}^0P = ({}^0R_1)^{-1} \, {}^0P_{before}$$

$$^1P_{new} = R(z,\theta) \, {}^1P_{before}$$

$$= R(z,\theta) \, ({}^0R_1)^{-1} \, {}^0P_{before}$$

$$^0P_{new} = {}^0R_1 \, {}^1P_{new}$$

$$^0P_{new} = {}^0R_1 \, \underline{R(z,\theta)} \, ({}^0R_1)^{-1} \, {}^0P_{before}$$

Similarty Transformation

$$R = {}^0R_1 \; \underline{R(z,\theta)} \; ({}^0R_1)^{-1}$$

لأي عملية حتى لو مش Rotation حول frame غير الـ frame الأساسي

$$^0R_1 = R(z,\alpha) \, R(y,\beta) \leftarrow$$

$$(R_1 R_2)^{-1} = R_2^{-1} \, R_1^{-1}$$

$$({}^0R_1)^{-1} = R^{-1}(y,\beta) \, R^{-1}(z,\alpha)$$

$$= R(y,-\beta) \, R(z,-\alpha) \leftarrow$$

$$R = R(z,\alpha) \, R(y,\beta) \, R(z,\theta) \, R(y,-\beta) \, R(z,-\alpha)$$

$$\tan\alpha = \frac{K_y}{K_x} \rightarrow \alpha$$

$$\tan\beta = \frac{\sqrt{K_x^2 + K_y^2}}{K_z} \longrightarrow \beta$$

given : $\vec{K} = (K_x, K_y, K_z)$ and $\theta$

we can Find the rotation ^_^

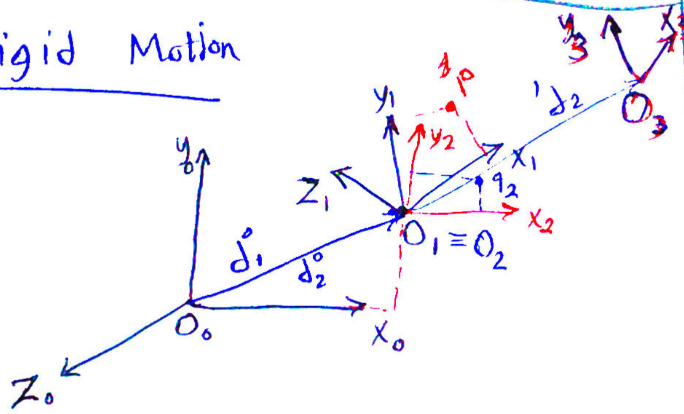
$$R = \begin{bmatrix} k_x^2(1-C_\theta)+C_\theta & - & - \\ - & k_y^2(1-C_\theta)+C_\theta & - \\ - & - & k_z^2(1-C_\theta)+C_\theta \end{bmatrix}$$

$$r_{11}+r_{22}+r_{33} = 1 - C_\theta + 3C_\theta$$

$$= 1 + 2C_\theta$$

$$\boxed{C_\theta = \frac{r_{11}+r_{22}+r_3 - 1}{2}}$$

## Rigid Motion




Frame 1 with respect to Frame 0

1- Translation $T(a, b, c)$

2- Rotation $R(\alpha, \beta, \gamma)$

given $^1p$ required $^0p$

$$^0p = d_1^0 + {^1p} \quad (\text{wrong } X)$$

$$^0q = d_2^0 + {^2q} \quad (\text{Right } \checkmark)$$

$$^0p = {^0d_1} + {^2p} \quad \checkmark$$

$$= {^0d_1} + {^2R_1}\,{^1p}$$

$$= {^0d_1} + {^0R_1}\,{^1p}$$

1- distance between $O_1$ and $O_0$ with respect to frame 1

2- Orientation of Frame 1 with respect to Frame 0 $\boxed{^0R_1}$

$$^0p = H\,{^1p}$$

$$^1p = H^{-1}\,{^0p}$$

H : homogenous Transformation Matrix

$$\begin{bmatrix} ^0p \\ 1 \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} ^0R_1 & ^0d_1 \\ \hline 0 & 1 \end{array}\right]}_{4\times4} \underbrace{\begin{bmatrix} ^1p \\ \hline 1 \end{bmatrix}}_{4\times1}$$

$$^0H_3 = {^0H_1}\,{^1H_3}$$

$$= \begin{bmatrix} ^0R_1 & ^0d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^1R_3 & ^1d_3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ^0R_3 & ^0R_1\,{^1d_3} + {^0d_1} \\ 0 & 1 \end{bmatrix}$$

$$H^{-1} \neq H^T$$

$$H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}$$

**Report** : Example 3.14 $\boxed{\text{Pg } 63}$

Example 3.13